INSAR PHASE UNWRAPPING BASED ON A COMBINATION OF MARKOV RANDOM FIELDS AND HYPERGEOMETRIC PHASE PDF MODELS

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1. INTRODUCTION

Phase unwrapping is a key step in extracting digital elevation models (DEMs) from interferometric synthetic aperture radar (InSAR) data. It consists in the reconstruction of the absolute phase values from the knowledge of its measured values modulus 2π. The most intuitive 1D-phase-unwrapping method consists in checking the absolute phase-differences values at adjacent sites and if it exceeds π then an integer multiple of 2π must be added or subtracted in order to make it less then π. This method can be extended to the 2D case by applying the above 1D process independently for each row and for each column. The problem here is that, since the row and column correction integers are chosen independently, the resulting phase differences may be inconsistent, especially for noisy phase fields or fields that present missing or unreliable phase information. Therefore, the unwrapped field which is obtained by the integration of the raw and column phase-differences values depends on the integration path and is in general incorrect [1]. In order to handle the problem of the inconsistencies, several methods were proposed in the literature that can be classified into to groups: the first one consists in integrating the difference fields in the consistent regions while cutting the image around regions where the difference field is inconsistent. The second consist in defining a single difference field close to the row difference field and to the column difference field in a mean-square sense. However, these methods are incapable of filtering out noise, interpolate the phase field across regions with invalid information and cannot handle discontinuities, since they have been designed to handle only the path dependent inconsistencies of the difference field [2]. In this work a general framework is presented to design algorithms for 2D, path independent phase unwrapping of locally inconsistent, noisy measured values phase field and that may contain regions of invalid information.

2. METHODOLOGY

This framework is based in Bayesian estimation theory with the use of Markov random field models to construct the prior distribution. Thus, the unwrapping problem solution is characterized as the minimizer of a phase-PDF-weighted quadratic function. Let adopt the following probabilistic framework: assume that the observed phase field g in the interval of [-π,π] and from which the absolute phase ϕ must be recovered, is given at a particular pixel and is equal to an arbitrary constant q plus the absolute phase ϕ corrected by the integer k plus a zero mean normal error:

$$g_i = \phi_i - 2\pi k_i + n_i + q \qquad i \in S, \tag{1}$$

where S denotes subset of pixels where one has valid information. The constant q may be eliminated by the definition of a new parameter f as:

$$f_i = k_i + \frac{q}{2\pi}. \tag{2}$$

The conditional probability of the observed phase g and f given ϕ, then is

$$P_{g,f/\phi}(\phi, f) = \frac{1}{K}\exp\left[-\alpha\sum_{i\in S}(g_i - \phi_i + 2\pi f_i)^2\right], \tag{3}$$

K is a normalizing constant and α is a parameter inversely proportional to the noise variance. Now the prior constraints on the behavior of the field ϕ are expressed on the form of a Markov random field model and the prior probability for a given field ϕ is given by the Gibbs distribution. The posterior distribution $P_{\phi,f/g}$ is obtained from Bayes rule:

$$P_{\phi,f/g}(\phi) = \frac{P_{g,f/\phi}(\phi, f)P_\phi(\phi)}{P_g(g)} = \frac{1}{Z_p}\exp[-U(\phi, f)], \tag{4}$$

where Z_P is a normalizing constant and the posterior energy $-U(\phi, f)$ is given by:

$$U(\phi, f) = \sum_C V_C(\phi) + \gamma \sum_{i \in S} (g_i - \phi_i + 2_i)^2. \tag{5}$$

The maximum a posteriori estimator for ϕ can be obtained by minimization of the energy with respect to ϕ and f. Following [3], the problem may be simplified by relaxing some constraints on the variable f and allow it to absorb noise and interpolate missing data. They proposed the following expression for the energy term:

$$U(f) = \sum_C V_C(g + 2\pi f) + \lambda \sum_{i,j} [f_i - f_j - r(f_i - f - j)]^2, \tag{6}$$

where $< i, j >$ denotes all nearest neighbor pairs of pixels. In this work the retained minimization algorithm is the gradient descent method throw the implementation of the following automata:

$$f_i^0 = \frac{g_i}{2\pi}, \quad f_i^{t+1} = f_i{}^t - h \frac{\partial U(f^{(t)})}{\partial f_i}, \tag{7}$$

where h is a small constant. A global smoothness may be enforced by quadratic potentials:

$$V_{i,j}(z_i, z_j) = (z_i - z_j)^2. \tag{8}$$

Such a potential function makes this algorithm work well for smooth surface. However it fails to recover discontinuous surfaces as well. Our main contribution in this work is to propose a modification to the above prior potential so that the jumps contribution to the energy are mitigated by weighting them by the probability of their occurrence which decrease when they increase, the proposed potential function is then:

$$V_{i,j}(z_i, z_j) = (z_i - z_j)^2 P_Z(z_i - z_j). \tag{9}$$

In our case, Z corresponds to the absolute phase ϕ which pdf can be written as an expression implying hypergeometric functions. However, A good approximation can be given by a Gaussian model:

$$P_\Phi(\phi) = \frac{1}{\sqrt{2\pi}\sigma_\phi} \exp(-\frac{(\phi - \widehat{\phi})^2}{\sigma_\phi^2}), \tag{10}$$

where σ_ϕ and $\widehat{\phi}$ are respectively the variance and the mean of the absolute phase ϕ. Theses parameters can be easily estimated from the theoretical hypergeometric PDF. By considering this approximation it is obvious that the phase difference Z between two neighbors sites i and j follows a zero mean Gaussian distribution with a variance of $2\sigma_\phi$

$$P_Z(z_i - z - j) = \frac{1}{\sqrt{2\pi\sigma_\phi}} \exp(-\frac{(z_i - z_j)^2}{4\sigma_\phi^2}), \tag{11}$$

The gradient descent automation in equation 6 is then :

$$\begin{aligned} f_i^{t+1} \quad = \quad & f_i^t - \tfrac{\sqrt{2\pi}h}{\sigma_\phi} \times \textstyle\sum_{j \in N_i} \left[[g_i - g_j + 2\pi(f_i^t - f_j^t)] \exp \tfrac{[g_i - g_j + 2\pi(f_i^t - f_j^t)]^2}{4\sigma^2} \right] \\ & \times \left[1 - \tfrac{1}{4\sigma_\phi^2} [g_i - g_j + 2\pi(f_i^t - f_j^t)] \right] + \lambda [f_i^t - f_j^t - r(f_i^t - f_j^t)] \end{aligned} , \tag{12}$$

where N_i is the set of neighbors of the considered site i.

3. CENTRAL CONCLUSIONS

Results from simulations curried on synthetic data show that, as expected, the proposed algorithm performs well with noisy principal-value phase fields even if they contain regions of invalid information and succeeds in recovering discontinuous surfaces.

4. REFERENCES

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